Computers & Industrial Engineering 160 (2021) 107606

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Modeling the diffusion of generation products in the presence of heterogeneous strategic customers for determining optimal marketing-mix strategies

Somayeh Najafi-Ghobadi [a], Jafar Bagherinejad [a,*], Ata Allah Taleizadeh [b]

[a] Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran
[b] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

ARTICLE INFO

Keywords:
Diffusion model
Generation products
Marketing-mix variables
Heterogeneous strategic customers

ABSTRACT

It is well-known that customers' strategic behaviors can influence the diffusion of new products and, consequently, firms' profit. Thus, firms should consider the strategic behaviors of customers for making their marketing decisions. In this paper, we proposed a new diffusion model for generation products by considering heterogeneous strategic customers and two key marketing variables of pricing and advertising. It was shown that optimal pricing trajectory is a concave curve and there is a threshold for the optimal advertising trajectory. When customers are strategic, firms' optimal response is to decrease the advertising expenditure and price. We found that the advertising expenditure increases with increasing the discount rate, number of population size, and effectiveness of advertising. Also, firms should reduce the advertising expenditure when the impact of word-of-mouth advertising is high. Among other findings, it was shown that discount rate has a negative effect on the price. Further, higher advertising effectiveness could lead to higher price.

1. Introduction

With increasing global competition, firms continuously try to update their products by incorporating new features into them. In this line, if a new product is successful in the market, they will substitute it by new generations with updated usability, technologies, and appearance. This strategy enables firms to extend the product life cycle from a single product to a series of products and to decrease the length of the development time. We are surrounded by many multi-generation products. For instance, firms such as Apple, Samsung, Blackberry, Motorola, and Nokia launch a new generation of their products almost every year.

One of the challenges of firms in managing multi-generation products is to predict the demand for each generation. A correct prediction plays a crucial role in strategic planning and policy definition for new products. In the literature of new product development management, diffusion models are well-known for understanding and predicting the demand. In addition to demand forecasting, these models also provide information about the acceptance speed of the new product by customers (Rogers, 1983; Mahajan et al., 1995; Kapur et al., 2017). Studies on diffusion models and their applications have increased with increasing competition in the markets (Lee & Huh, 2017). Following seminal research by Bass (1969), researchers have paid attention to propose diffusion models with considering marketing mix-variables such as pricing and advertising (Yenipazarli, 2015; Dhakal et al., 2019). In all of the classical diffusion models for new products, it is assumed that the diffusion of new products in the market follows the bell shape curve. In this growth pattern, sales slowly increase initially and, then, rise rapidly until the maturity level; by decreasing the potential customers, they decrease until reaching zero.

Nowadays, due to advances in the Internet, customers can obtain valuable information about new generations. Websites such as Decide.com and MacRumors.com provide helpful information about the coming products. For example, long before Apple launched iPhone X and iPhone 8 to the market, customers had been introduced with new features and attributes of these products at MacRumors.com. Also, this experience was repeated for Apple Watch, iPod, and MacBook. Forecasting the new generation's arrival, the customers of the old generations may behave strategically and decide to postpone their purchase to the newer one. A strategic customer does not make haste to buy the current generation and may wait for the newer one if he/she gains more surplus from buying the newer generation. The sale story of Apple's iPhone is an illustrative example of customers' strategic behaviors concerning the

* Corresponding author at: Alzahra University, North Sheikh Bahaee St., Deh-e-Vanak, Tehran, Iran.
E-mail addresses: s.najafi2010@gmail.com (S. Najafi-Ghobadi), jbagheri@Alzahra.ac.ir (J. Bagherinejad), Taleizadeh@ut.ac.ir (A.A. Taleizadeh).

https://doi.org/10.1016/j.cie.2021.107606
Received 9 August 2020; Received in revised form 28 May 2021; Accepted 2 August 2021
Available online 4 August 2021
0360-8352/

coming of a newer generation. The pre-order for iPhone 5 was more than two million just 24 h after it was released. Apple sold more than 13 million devices from iPhone 6 s and 6 s + on an opening day in 2015. This large number of demand revealed that customers were waiting for the arrival of these new generations to the market. Also, the sales of Apple's PC started to decline soon after they were launched. Customers' strategic behaviors have been seen in different industries (Liu et al., 2018; Lobel et al., 2015; Li et al., 2014). Researchers have reported that up to 70 percent of customers expose strategic behaviors in various industries (Osadchiy & Bendoly, 2015).

The above observations reveal that the bell shape curve models are potentially inapplicable to explain the diffusion of multi-generation products when customers are strategic. Considering the significance of this issue, this study was conducted to explore the following questions: (1) What is the diffusion of a new generation product in the presence of strategic customers? (2) What is the optimal pricing and advertising schemes for a new generation product when customers behave strategically? (3) How can manufacturers maximize the total profit of generation products in the presence of strategic customers? Answering these questions is important because researchers and practitioners are constantly seeking and updating their understanding of the diffusion of multi-generation products in order to accurately explain and predict their sales rate. An accurate forecasting sales rate helps the managers of generation products in their policy definition and strategic planning.

In this regard, we presented a new diffusion model by taking into account the strategic behavior of customers. We used our proposed diffusion model to optimize the profit of a firm managing a generation product. The firm aimed to determine the optimal price and advertising decisions for the first generation. It was supposed that potential customers were heterogeneous strategic customers. They did not necessarily purchase the first generation even if they gained a positive surplus. Strategic customers might delay the purchase to the second generation if they have a more surplus. The Pontryagin's maximum principle was used to analyze the proposed model.

The remainder of this study is organized as follows: we survey the related studies in Section 2. In Section 3, the problem is described and the mathematical formulation is presented. The optimal analysis and a heuristic algorithm are proposed in Sections 4 and 5, respectively. In Section 6, numerical examples are solved and the sensitivity analysis is carried out. Finally, we outline the key results and provide future research opportunities in Section 7.

2. Related studies

Bass proposed the pioneering and most successful diffusion model in 1969, describing the bell-shaped curve of new products' diffusion. According to his model, there are two kinds of customers: innovators and imitators. Innovative customers purchase the product based on their personal preference, while imitative customers purchase the product based on the recommendations of those who have bought the product (Bass, 1969). By using the hazard rate function and considering a fixed number of potential customers (N), Bass modeled the new products acceptance rate (*f(t)*) in the market as follows:

$$f(t) = \frac{\partial F(t)}{\partial t} = (\phi + \varphi\frac{F(t)}{N})(N - F(t)) \tag{1}$$

where F(t) is the cumulative number of customers that buy the new product at time t and$(N - F(t))$ is the number of remaning customers at the market at time t. ϕ and φ are the coefficient for innovation and imitation, respectively. In the Bass model (Equation (1)), the numbers of innovative and immitative customers that buy the product at time t are $\phi(N - F(t))$ and $\varphi\frac{F(t)}{N}(N - F(t))$, respectively.

Despite the strengths of the Bass model in predicting the sales rate of new products (Guidolin, 2008; Wright et al., 1997; Adams, 2004; Homer, 1987), this model suffers from some limitations. As an example, it views the spread of new products as communication between innovators and imitators, and neglects the effect of marketing variables in the diffusion of new products in the market. Thus, subsequent studies have extended the Bass model via investigating the impact of pricing in different ways: some researchers (Bass & Bultez, 1982; Huang et al., 2007; Sethi et al., 2008; Şeref et al., 2016; Wu et al., 2017; Sale et al., 2017) have multiplied the Bass model to a function of price ($g(p(t))$) (Equation (2)); others (Mahajan & Peterson, 1978; Mahajan & Peterson, 1982; Feichtinger, 1982) have considered the number of potential customers as a function of price ($N(P(t))$) (Equation (3)).

$$f(t) = \frac{\partial F(t)}{\partial t} = (\phi + \varphi\frac{F(t)}{N})(N - F(t))g(p(t)) \tag{2}$$

$$f(t) = \frac{\partial F(t)}{\partial t} = (\phi + \varphi\frac{F(t)}{N(P(t))})(N(P(t)) - F(t)) \tag{3}$$

Another growing stream of the literature on new product development and marketing has analyzed the role of advertising in the Bass model. Bass used the social theory of communication and argued that the word-of-mouth advertising (WOM advertising) influences imitators. In examining the effect of advertising on the diffusion of a new product, Horsky & Simon (1983) reported that just innovators are affected by the firm's advertising. Dockner & Jorgensen (1988) investigated the impact of advertising in three situations: advertising influences either innovators or imitators, or both of them. Swami & Khairnar (2006) analyzed availability and awareness advertising. A dynamic advertising model was studied by Krishnan & Jain (2006) who indicated the effectiveness of advertising, ratio of advertisement to profits, and discount rate as the influential factors for the optimal advertising strategy. Swami & Dutta (2010) proposed a decreasing pricing strategy for new durable products in emerging markets. They applied the proposed model to the electronic product of Japan. Yenipazarli (2015) dealt with pricing, advertising, and warranty problems for new products and used Pontryagin's maximum principle to analyze his model.

Nowadays, firms are always launching new generations of products with improved features. Each successive substitution can increase the demand by creating a new potential market and attracting previous adopters to update their purchase. Also, the newer generation may plunder the customers of the older generation due to its advanced attributes. In other words, potential customers of the older generation may behave strategically and delay their purchase to the next generation via forecasting the introduction of the new generation. In the first try, Shi et al. (2014) proposed a diffusion model by considering customers' strategic behaviors. The duration of the product presence in the market was considered as a factor affecting strategic customers' behaviors. They considered a monopoly market with homogeneous strategic customers. The proposed diffusion model was tested by demand data of home entertainment, mobile phones, as well as audio and computing industries. Findings revealed that the new diffusion model forecasted the sales more accurately than the older ones. In the literature of strategic behaviors, the price of a new product has been highlighted as the most influential factor for the customers' strategic behaviors. For more details about these works, the reader can study Aviv & Pazgal (2008), Tang & Netessine (2009), Zhou & Wu (2011), Swinney (2011), Özer et al. (2012), Gönsch et al. (2013), Liang et al.(2014), Chen & Chen (2015), Zhou et al. (2015), Wei & Zhang (2018), Liu et al. (2019), Liu et al. (2020) and Chen & Trichakis (2021). In this line, a diffusion model for a two-generation product by considering pricing was formulated by Guo & Chen (2018). They assumed that the price of both generations was equal and customers were heterogeneous in evaluating both generations. The pricing decision was analyzed numerically. We sum up the reviewed studies in Table 1.

All studies explained above are largely successful attempts in providing a model to manage the new products, yet they have three major deficiencies as follows:

Table 1
Summery of reviewed studies.

Use an optimization solution method		Type of strategic customer		Customers' behavior	Decision variable		Propose a diffusion model		Study
No	Yes	Homogeneous	Heterogeneous		Advertising	Pricing	**No**	**Yes**	
*				Myopic				*	Bass (1969)
	*			Myopic		*		*	Mahajan & Peterson (1978)
	*			Myopic		*		*	Bass & Bultez (1982)
	*			Myopic		*		*	Mahajan & Peterson (1982)
	*			Myopic		*		*	Feichtinger (1982)
	*			Myopic	*			*	Horsky & Simon (1983)
	*			Myopic	*			*	Dockner & Jorgensen (1988)
	*			Myopic	*	*		*	Swami & Khairnar (2006)
	*			Myopic	*			*	Krishnan & Jain (2006)
	*			Myopic		*		*	Huang et al (2007)
				Strategic		*	*		Aviv and Pazgal (2008)
	*			Myopic	*	*		*	Sethi et al., (2008)
	*			Myopic	*			*	Swami & Dutta (2010)
	*	*		Strategic		*	*		Swinney (2011)
*		*		Strategic				*	Shi et al. (2014)
	*	*		Strategic		*	*		Liang et al.(2014)
	*			Myopic	*	*		*	Yenipazarli (2015)
	*	*		Strategic		*	*		Zhou et al. (2015)
	*			Myopic		*		*	Şeref et al. (2016)
	*			Myopic		*		*	Wu et al. (2017)
	*			Myopic		*		*	Sale et al. (2017)
*			*	Strategic		*		*	Guo & Chen (2018)
	*		*	Strategic		*	*		Liu et al. (2019)
	*		*	Strategic		*	*		Liu et al. (2020)
	*		*	Strategic		*	*		Chen & Trichakis (2021)
	*		*	**Strategic**	*	*		*	**The present study**

1. The effect of advertising expenditure, despite the point that many marketing researchers have highlighted its effectiveness in the success of new products, has not been investigated in the market populated with strategic customers.
2. Prior research on joint pricing and advertising decisions has considered myopic customers.
3. Previous studies have just proposed a diffusion model for generation products in the presence of strategic customers and have not used optimization solution methods to analyze their model analytically.

In an effort to address these limitations, we make four contributions to the literature. First, we provide a novel diffusion model for generation products by considering the effect of pricing, advertising, and strategic behaviors of customers, simultaneously. Second, we assume that customers are heterogeneous in the valuation of the new generation product. Third, we use the Pontryagin's maximum principle to analyze the proposed model analytically and to find the optimal schemes of pricing and advertising decisions. Fourth, we provide an efficient heuristic to solve numerical examples and conduct a sensitivity analysis.

3. The suggested model

In this research, we consider a firm that launches a two-generation durable product to a monopoly market. The firm releases the first generation (P_1) at time 0 and, after its life cycle, introduces the new generation (P_2) at time T. In industries such as high-technology products and fashion apparel, generation products abound. The firm wants to determine the optimal price ($W_1(t)$) and advertising expenditure ($U_1(t)$) for P_1 over its life cycle by maximizing the total discounted profit. The potential customers of P_1 can anticipate P_2 and their purchase decision is strategic. Because of the advanced attributes of P_2, they may not buy P_1 and wait for the new generation. Thus, all customers that become aware of P_1 through advertising will not buy it. They compare the surpluses that they can get from buying P_1 and P_2 and, then, decide whether to buy P_1 or wait for P_2. Therefore, the customers' purchase decision will be classified into two phases: the awareness phase and the adoption phase.

3.1. Awareness phase

Advertising acquaints customers with P_1. We are inspired by the Bass diffusion model to model the number of aware customers at time t. Thus, the potential customers of P_1 are classified into two groups: innovative customers and imitative customers. Researches have revealed that up to 90% of advertising expenditures are expended on innovators to initially persuade them to buy a new product (Horsky & Simon, 1983; Swami & Dutta, 2010; Yenipazarli, 2015). They show that if innovative customers buy the new product in the early stages, imitative customers will be attracted by WOM advertising in later stages and the new product acceptance will be speeded up in the market. We also suppose that the firm's advertising and WOM advertising affect innovative and imitative customers, respectively. Let N be the number of the population size. With defining Y(t) as the cumulative number of aware customers at time t, the awareness rate of P_1 at time t is:

$$y(t) = \frac{\partial Y(t)}{\partial t} = \left(\phi + \beta Ln(U_1(t)) + \varphi\frac{Y(t)}{N}\right)(N - Y(t)) \qquad 0 < t < T \tag{4}$$

where φ and ϕ are the coefficient of imitation and innovation, respectively. Equation (4) represents the number of customers that become aware of P_1 through firm's advertising or WOM advertising at time t. In this equation, the customers' awareness rate at time t is proportionate to: (1) the remaining potential customers represented by $(N - Y(t))$, (2) the influence of the firm's advertising denoted by $(\phi + \beta Ln(U_1(t)))$, and (3) the influence of WOM advertising presented by $(\varphi\frac{Y(t)}{N})$.

3.2. Adoption phase

When customers become aware of P_1 through WOM advertising or the firm's advertising at time t, they choose to purchase it or not. They compute surpluses of purchase P_1 and P_2, and then, decide about their purchase. We assume that aware customers are heterogeneous in the valuation of P_1 and P_2. Thus, the aware customer's taste for the product value (η) is considered as a random variable. We denote the customer's evaluation of P_1 and P_2 by ηV_1 and ηV_2, respectively. By defining the customers' surplus as the difference between customers' evaluation of the product and the product price (Liu et al., 2019; Dong & Wu, 2019), the aware customer's surplus from purchasing P_1 at time t will be:

$$\eta V_1 - W_1(t) \qquad 0 < t < T \tag{5}$$

We suppose that the price of P_2 at time T ($W_2(T)$) is pre-announced by the firm at the beginning of the planning horizon. In the literature, the pre-announcing strategy is proposed as a proper strategy to prevent strategic customers from delaying the purchase (Elmaghraby et al., 2008; Elmaghraby et al., 2009; Cachon & Feldman, 2015; Shum et al., 2017; Dong & Wu, 2019; Liu et al., 2019). Avive & Pazgal (2008) reported that pre-announcing strategy increases the firms' profit up to 8.32%. In practice, Apple used this strategy for selling its iPhone series. Sony pre-announced the price of PS4. Thus, if the aware customer decides to wait for P_2, he/she earns a surplus as follows:

$$\alpha(\eta V_2 - W_2(T)) \tag{6}$$

The difference between customers' evaluation of P_2 and the price of P_2 multiplied by α is considered as the aware customers' surplus from delaying purchase to P_2. Parameter α ($\alpha \in [0,1]$) shows the level of customers' strategic behaviors. With increasing α, the strategic behavior of customers increases. When $\alpha = 0$, customers are myopic and they do not consider the surplus of buying P_2.

The aware customers choose to buy P_1 at time t when they have a positive surplus that is more than waiting for P_2, that is:

$$\eta V_1 - W_1(t) > \alpha(\eta V_2 - W_2(T))^{+} \qquad 0 < t < T \tag{7}$$

In other words, they purchase P_1 at time t if their taste in the product value is:

$$\eta > \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2} \qquad 0 < t < T \tag{8}$$

We assume that η follows uniform distribution within [0, 1]. Therefore, the probability by which the aware customers purchase P_1 at time t will be:

$$1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2} \qquad 0 < t < T \tag{9}$$

By considering the purchase probability of the aware customers in Equation (9) and the number of aware customers in Equation (4), the demand rate (diffusion speed) for P_1 at time t will be:

$$s(t) = \frac{\partial S(t)}{t} = \left(\phi + \beta ln(U_1(t)) + \varphi\frac{S(t)}{N}\right)(N - S(t))\left(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2}\right) \quad 0 < t < T$$

where S(t) is the cumulative demand for P_1 at time t. It is noticeable that previous adopters of P_1 can affect other potential customers by WOM advertising.

The objective function is defined as the total discounted profit of the firm from selling P_1 in Equation (10). Parameters C and ρ are the production cost and discount rate, respectively.

$$\underset{W_1(t),U_1(t)\geqslant 0}{Max} \pi = \int_0^T e^{-\rho t}[(W_1(t) - C)s(t) - U_1(t)]dt \tag{10}$$

Subject to

$$s(t) = \left(\phi + \beta ln(U_1(t)) + \varphi\frac{S(t)}{N}\right)(N - S(t))\left(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2}\right) \quad 0 < t < T \tag{11}$$

$$S(0) = 0 \tag{12}$$

The demand for P_1 at time t is presented in Equation (11). The initial demand of P_1 at time 0 is zero and shown in Equation (12).

4. Optimal solution

In this study, we apply Pontryagin's maximum principle to solve the proposed model. The price and advertising expenditure at time t ($W_1(t)$ and $U_1(t)$) are control variables, and the cumulative demand at time t ($S(t)$) is the state variable. Assuming $\lambda(t)$ as the adjoint variable, the Hamilton-Jacobi-Bellman function is as follows:

$$H(t) = (W_1(t) - C + \lambda(t))(\phi + \beta ln(U_1(t)) + \varphi\frac{S(t)}{N})(N - S(t))(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2}) - U_1(t) \tag{13}$$

This Hamilton-Jacobi-Bellman (13) is the sum of the current profit and future benefit, and illustrates the instantaneous total discounted profit of the firm at time t. Based on Equation (13) and Pontryagin's maximum principle (the reader can find the complete description of Pontryagin's maximum principle and its necessary conditions in Sethi & Thompson (2000)), the necessary conditions of optimality are:

$$\begin{aligned}\frac{\partial\lambda(t)}{\partial t} &= \rho\lambda(t) - \frac{\partial H(t)}{\partial S(t)}\\ &= \rho\lambda(t) - (W_1(t) - C + \lambda(t))(\varphi - \phi - \beta Ln(U_1(t)) - 2\varphi\frac{S(t)}{N})(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})\end{aligned} \tag{14}$$

With the boundary condition$\lambda(T) = 0$.

Also, we have:

$$\frac{\partial H(t)}{\partial W_1(t)} = -(W_1(t) - C + \lambda(t))(\frac{1}{V_1 - \alpha V_2}) + (1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2}) = 0 \tag{15}$$

$$\frac{\partial H(t)}{\partial U_1(t)} = \frac{\beta}{U_1(t)}(W_1(t) - C + \lambda(t))(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})(N - S(t)) - 1 = 0 \tag{16}$$

The second derivatives of Equation (13) are equal to:

$$\frac{\partial^2 H(t)}{\partial W_1^2(t)} = -\frac{2}{V_1 - \alpha V_2} < 0 \tag{17}$$

$$\begin{aligned}\frac{\partial^2 H(t)}{\partial U_1^2(t)} &= -\frac{\beta}{(U_1(t))^2}(W_1(t) - C + \lambda(t))(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})(N - S(t))\\ &= -\frac{1}{U_1(t)} < 0\end{aligned} \tag{18}$$

And

$$\frac{\partial^2 H(t)}{\partial W_1(t)\partial U_1(t)} = 0 \tag{19}$$

Therefore, we obtain:

$$H_{W_1(t)W_1(t)}H_{U_1(t)U_1(t)} - (H_{U_1(t)W_1(t)})^2 = \frac{2}{U_1(t)(V_1 - \alpha V_2)} > 0 \tag{20}$$

Thus, the solutions obtained from Equations (15) and (16) are the optimal solutions for the proposed discounted profit function.

4.1. Analysis of pricing and advertising expenditure trajectories

In this section, the optimal trajectories of pricing and advertising expenditure are analyzed in Theorems 1 and 2 (the dot notation represents the time derivative).

Theorem 1:. *The optimal pricing trajectory of P_1 is a concave curve, the* *maximum of which occurs at* $t^* \in (0, T)$, *such that* $W_1(t^*) = -\frac{\rho\lambda(t^*)}{s_{S(t^*)}} + V_1 - \alpha(V_2 - W_2(T))$.

Proof:. *See Appendix A.*

According to Theorem 1, the optimal trajectory of pricing is a concave curve. It is obvious that the maximum of $W_1(t)$ reduces with increasing customers' strategic behavior (α) and discount rate (ρ).

Theorem 2:. *There is a threshold* $\overline{U} = \frac{N\beta\rho\lambda(t)}{\varphi}$ *for the optimal advertising expenditure trajectory* ($U_1(t)$) *at any time* $t \in (0, T)$, *so that:*

i. $U_1(t) < 0$ if $U_1(t) > \overline{U}$
ii. $U_1(t) = 0$ if $U_1(t) = \overline{U}$
iii. $U_1(t) > 0$ if $U_1(t) < \overline{U}$

Proof:. *See Appendix B.*

5. Heuristic algorithm

According to Equations (14)-(16), the decision variables of our proposed model are correlated. Thus, a heuristic algorithm is needed to solve numerical examples. We write the discrete version of the proposed model by using Rosen's discretization technique (Rosen, 1968) (see Appendix C). The discrete version of Pontryagin's maximum principle is used and a heuristic algorithm is provided as follows:

i. Considering initial values $W_1^n = C$ and $U_1^n = 1$ for n = 1, 2…T-1.

ii. Computing the differential equations (ΔS^n) forward by considering $S^0 = 0$

$$s^n = \Delta S^n = \left(\phi + \beta ln\left(U_1^n\right) + \varphi\frac{S^n}{N}\right)(N - S^n)\left(1 - \frac{W_1^n - \alpha W_2^T}{V_1 - \alpha V_2}\right)$$
$$n = 1, 2 \ldots T-1$$

iii Calculating the differential equations ($\Delta\lambda^n$) backward by assuming $\lambda^T = 0$

$$\Delta\lambda^n = \rho\lambda^{n+1} - \left(W_1^n - C + \lambda^{n+1}\right)\left(\varphi - \phi - \beta Ln\left(U_1^n\right) - 2\varphi\frac{S^{n-1}}{N}\right)\left(1 - \frac{W_1^n - \alpha W_2^T}{V_1 - \alpha V_2}\right)$$
$$n = 1, 2 \ldots T-1$$

Theorem 2 reveals it is optimal to decrease/ keep steady/ increase the adverting expenditure ($U_1(t)$) at t ∈ (0, T) if $U_1(t) > \overline{U}$, $U_1(t) = \overline{U}$ or $U_1(t) < \overline{U}$. Threshold $\overline{U}$ shifts up with increasing advertising effectiveness (β), discount rate (ρ), and population size (N), and shifts down with increasing the effect of WOM advertising (φ).

iv Updating W_1^n and U_1^n by utilizing

$$W_1^n = \frac{1}{2}\left(C - \lambda^{n+1} + V_1 - \alpha V_2 + \alpha W_2^T\right)$$
$$n = 1, 2 \ldots T-1$$

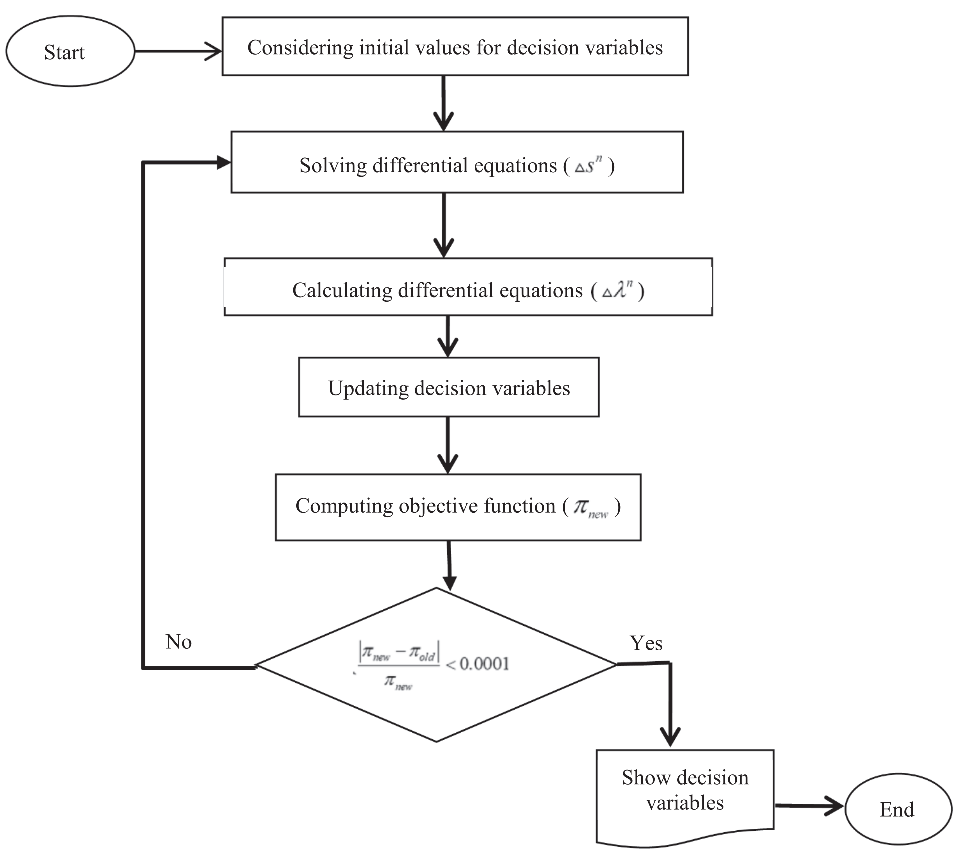

Fig. 1. Flowchart of the proposed heuristic algorithm.

$$U_1^n = \beta\left(W_1^n - C + \lambda_1^{n+1}\right)\left(N - S^{n-1}\right)\left(1 - \frac{W_1^n - \alpha W_2^T}{V_1 - \alpha V_2}\right)$$
$$n = 1,\ 2, ..., T - 1$$

v. Calculating the objective function (π_{new}).

vi. Checking $\frac{|\pi_{new} - \pi_{old}|}{\pi_{new}} < 0.0001$. If it was valid, stop; otherwise, go back to ii.

The flowchart of the proposed heuristic solution algorithm is presented in Fig. 1.

6. Computational results

To gain further insight into the analytical results, first, we present a numerical example and, then, sensitivity analysis concerning main parameters is carried out. This study can be utilized to investigate the pricing and advertising strategies of new generation products in various industries. We consider the cell phone industry to conduct the numerical study. Nowadays, firms in this industry such as Apple, Samsung, Blackberry, Motorola, and Nokia launch a new generation of their products almost every year. Due to introducing successive generations in the cell phone industry, the customers' strategic behavior is quite evident in this market. For example, we can mention the significant impact of customers' strategic behavior in the sale of Apple's iPhone. We consider a monopolist firm that introduces a new generation product for 12 months (T = 13) to a market with 30,000 potential customers. The customers' strategic behavior coefficient is 0.5. Reports show that φ + ϕ changes from 0.2 to 1 for most of the new products (Krishnan et al., 1999). Thus, we assume ϕ = 0.1 and φ = 0.4. According to the cell phone industry, we set $\beta = 0.03$, $V_1 = 800$, $V_2 = 900$, $W_2(T) = 600$, $C = 450$, and $\rho = 0.3$. We code the proposed algorithm in MATLAB software and use an Intel Core i7 – 2.5 GHz to 3.1 GHz laptop with 8.00 GB of RAM to solve the numerical examples.

It can be seen that the numerical example (Table 2) demonstrates the results of optimal pricing and advertising strategies shown in Theorems 1 and 2. The firm allocates a high advertising expenditure to promote P_1 in the first period (18848.77). The advertising expenditure is decreasing because it perches above $\overline{U}$. The pricing trajectory is a concave curve. The price increases until period seven and declines afterwards.

6.1. Sensitivity analysis

We investigate the impact of changes in the values of the main parameters of the proposed diffusion model on the objective function as well as optimal pricing and advertising strategies according to Table 3. The numerical example in the previous section is used and the following results are achieved.

6.1.1. Strategic behavior coefficient

In the proposed diffusion model, α shows the strategic behavior of customers. With increasing α, the customers' desire to wait for the new generation will increase. Numerical examples reveal that α has a negative effect on the price of P_1 Table 4.. A higher customers' strategic behavior induces the firm to determine a lower price for P_1. The price of P_1 decreases from 637.31 to 518.48 in the first period when α changes from 0 to 0.8. Also, the advertising expenditure is shown to be negatively affected by customers' strategic behavior. Firms should reduce the advertising expenditure in the markets with strategic customers. The amount of reduction in advertising expenditure is higher in the early life of P_1. Fig. 2 shows that the firm's profit decreases with increasing α. It declines from 1117646.3 to 811495.40. From Fig. 2, we can conclude that the firm's maximum profit will be achieved in the situation that customers are myopic (α = 0) or strategic enough (α = 0.8).

Table 2
Optimal pricing and advertising expenditure trajectories.

Period	$W_1^*(t)$	$U_1^*(t)$
1	559.65	18848.77
2	561.38	16156.9
3	562.84	13815.98
4	564.06	11791.47
5	565.02	10051.55
6	565.72	8567.725
7	566.09	7315.67
8	566	6276.56
9	565.19	5439.42
10	563.14	4805.31
11	558.82	4396.49
12	550	4278.54
Profit	784615.78	

Table 3
The range of changes in parameter values.

Parameter	Min	Max
α	0	0.8
β	0.01	0.09
φ	0.1	0.7
ρ	0.1	0.8
N	10,000	40,000

6.1.2. Advertising expenditure coefficient

In the proposed diffusion model, parameter β is used to present the impact of advertising expenditure. In Theorem 2, it is revealed that the advertising threshold increases with increasing β, which is also demonstrated by the findings of numerical examples. The firm expends more budget on advertising when it can be more successful in attracting customers through advertising. It is noticeable that the increase in advertising expenditure will be allocated to initiation periods, which is due to the point that innovative customers are influenced by direct advertising. When more innovators attract to P_1 and buy it, the WOM advertising will be increased in subsequent periods. A higher advertising expenditure leads to a higher price (Table 5). As expected, the firms' profit increases with increasing β (Fig. 3).

6.1.3. WOM advertising coefficient

WOM advertising has been highlighted as a well-known type of advertising. Recommending a new product by people who have already bought it makes customers more confident to buy the new product. It is revealed that the advertising expenditure threshold shifts down with increasing the WOM advertising coefficient (φ) in Theorem 1. In line with this, numerical examples represent there is a negative correlation between φ and advertising expenditure (Table 6). As coefficient φ increases, since imitators are influenced by WOM advertising, the firm reduces its advertising expenditure. On the other hand, by increasing φ, the firm enhances the price of P_1. By doing this strategy, the firms' profit will increase (Fig. 4).

6.1.4. Discount rate coefficient

Discount rate (ρ) is indicated in Theorems 1 and 2 as a key parameter influencing pricing and advertising strategies. The discount rate shows the degree of the firm's patience. In this section, we analyze the influence of this parameter by changing it from 0.1 to 0.8. Results exhibit that the firm should spend a higher advertising expenditure and set a lower price by increasing ρ (Table 7). The discount rate also has a negative impact on the firm's profit (Fig. 5). Therefore, the managers of new generation products should be more patient in the markets with strategic customers.

6.1.5. Population size

Theorem 2 shows that the advertising expenditure increases by the population size (N). We investigate this result at five values for population size, N = 10000, 20000, 30000, 40000, and 50000. Table 8

Table 4
Impact of the customers' strategic behavior on price and advertising expenditure.

Parameter	α = 0		α = 0.2		α = 0.4		α = 0.6		α = 0.8	
Period	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*
1	637.31	27314.92	605.99	23801.66	574.97	20427.92	544.68	17402.6	518.48	15475.1
2	639.53	24252.62	607.99	20968.75	576.76	17737.65	546.35	14594.08	520.21	10964.58
3	641.47	21492.68	609.72	18435.54	578.30	15367.12	547.75	12205.07	521.42	7715.24
4	643.12	19019.21	611.19	16183.03	579.60	13289.98	548.89	10183.93	522.24	5393.41
5	644.46	16816.92	612.38	14193.1	580.64	11481.5	549.79	8484.57	522.76	3751.34
6	645.44	14872.24	613.24	12449.44	581.39	9919.39	550.42	7066.39	523.06	2601.87
7	645.93	13174.8	613.69	10938.88	581.79	8584.85	550.76	5894.69	523.18	1804.64
8	645.76	11719.66	613.55	9653.296	581.68	7464.14	550.68	4941.6	523.08	1256.72
9	644.56	10510.77	612.51	8592.727	580.79	6551.24	549.95	4187.82	522.63	885.05
10	641.67	9566.98	609.97	7770.689	578.59	5852.52	548.05	3626.28	521.34	640.54
11	635.91	8933.58	604.85	7224.71	574.03	5396.27	543.90	3270.53	517.70	495.22
12	625	8706.95	595	7039.43	565	5254.99	535	3177.29	505	450.12

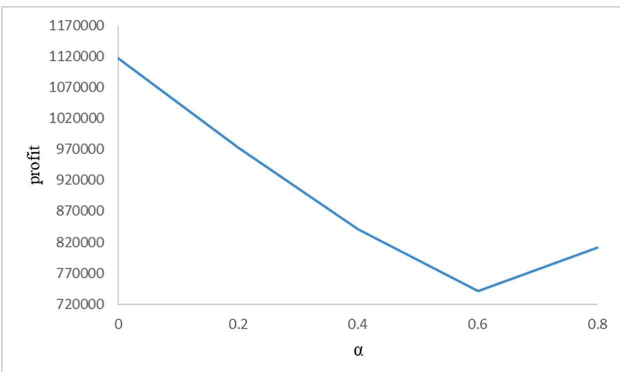

Fig. 2. Effect of customers' strategic behavior coefficient on profit.

reveals that a higher population size of the target market induces the firm to determine higher advertising expenditure, while the price is not very sensitive to the increase of N. As expected, the firm's profit increases with the target market population size (Fig. 6).

6.2. *Managerial insight*

Nowadays, the products' life cycle has been shortened under the rapid growth of technology in various industries. Hence, firms regularly substitute their successful products with newer generations that have better attributes and features. The successive introduction of new generations leads customers to behave strategically and postpone their purchases until the arrival of the new generation. Numerous pieces of evidence of customers' strategic behavior have been observed in selling the generation products. Therefore, firms should consider the strategic behavior of customers in their decision-making. In this research, we provide a new diffusion model to analyze the pricing and advertising decisions for generation products when customers are strategic. In summary, we gain the following managerial insights from this research:

1. This research prepares a guidance to help the manufactures of generation products to find an appropriate strategy about pricing and advertising decisions in the situations, in which customers behave strategically.
2. We extend the traditional diffusion models by considering pricing, advertising, and strategic customers. The proposed diffusion model enables generation product managers to accurately predict product sales over time by considering product penetration in the market.

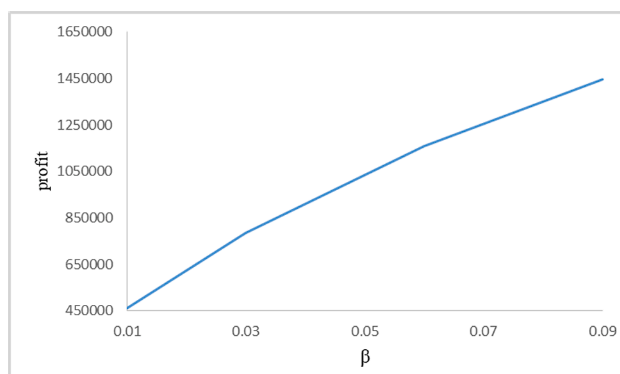

Fig. 3. Effect of the advertising expenditure coefficient on profit.

Table 5
Impact of the advertising expenditure coefficient on price and advertising expenditure.

Parameter	β = 0.01		β = 0.03		β = 0.06		β = 0.09	
Period	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*
1	548.65	8319.04	559.65	18848.77	569.51	27846.12	575.94	33037.8
2	550.52	7533.33	561.38	16156.9	570.86	22249.45	576.93	24787.9
3	552.23	6803.38	562.84	13815.98	571.90	17744.94	577.59	18584.44
4	553.75	6129.67	564.06	11791.47	572.67	14136.64	578	13939.95
5	555.08	5512.53	565.02	10051.55	573.20	11260.09	578.19	10474.34
6	556.19	4952.39	565.72	8567.72	573.49	8978.94	578.18	7895.40
7	557.012	4450.09	566.09	7315.67	573.49	7182.51	577.95	5982.40
8	557.46	4007.39	566	6276.56	573.06	5784.01	577.35	4572.20
9	557.38	3627.85	565.19	5439.42	571.87	4720.08	576.05	3548.76
10	556.50	3318.21	563.14	4805.31	569.21	3953.30	573.22	2837.26
11	554.33	3090.93	558.82	4396.49	563.37	3482.33	566.72	2407.27
12	550	2969.27	550	4278.54	550	3377.51	550	2304.37

Table 6
Impact of WOM advertising coefficient on price and advertising expenditure.

Parameter	φ = 0.1		φ = 0.3		φ = 0.5		φ = 0.7	
Period	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*
1	559.65	18848.77	566.43	15431.42	571.46	13072.22	575.43	11313.65
2	561.38	16156.9	568.01	12631.41	572.87	10252.23	576.70	8517.21
3	562.84	13815.98	569.28	10310.5	573.96	8013.872	577.63	6386.56
4	564.06	11791.47	570.28	8397.52	574.77	6248.19	578.29	4774.74
5	565.02	10051.55	571.02	6830.20	575.35	4863.83	578.72	3563.26
6	565.72	8567.72	571.51	5554.87	575.69	3785.26	578.96	2658.11
7	566.09	7315.67	571.68	4526.42	575.75	2951.27	578.96	1986.14
8	566	6276.56	571.40	3708.66	575.39	2313.76	578.59	1491.82
9	565.19	5439.42	570.34	3075.26	574.27	1837	577.50	1134.60
10	563.14	4805.31	567.82	2612.33	571.58	1498.13	574.82	887.34
11	558.82	4396.49	562.30	2325.28	565.35	1291.18	568.20	737.76
12	550	4278.54	550	2259.456	550	1243.23	550	699.77

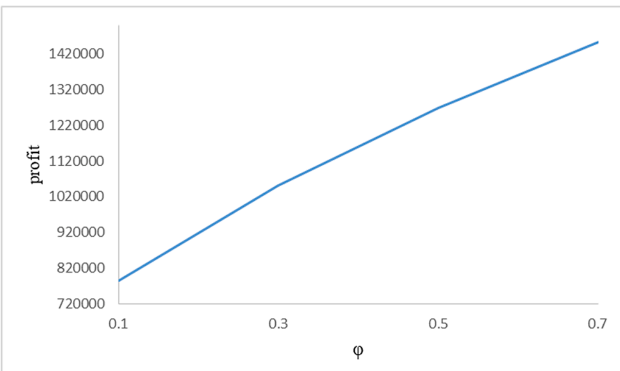

Fig. 4. Effect of WOM advertising coefficient on profit.

3. It would be better for managers of generation product with strategic customers to indicate a lower price and allocate a low advertising expenditure because it is more beneficial for them.
4. The findings of this research advise the managers to spend more on advertising along with higher prices in the market populated with strategic customers. This increase in advertising expenditure should be allocated more on initiation periods of the product life cycle. In this way, more customers will be attracted in initiation periods and the WOM advertising will be increased in later periods.
5. Furthermore, the results show that the WOM advertising can have the considerably positive impact on the profit. Thus, managers should try to improve the WOM advertising between customers. Designing websites and using social media facilitates such as Instagram can increase communication between customers to increase WOM advertising.
6. Managers should increase the advertising expenditure by increasing the discount rate, while they are advised to decrease the price.
7. Moreover, the analysis carried out on discount rate reveals this parameter has a significant effect on profit function. Increasing the discount rate leads to reduction in the firm's profit. Accordingly, managers should be more patient when customers are strategic.
8. The managers of generation products with strategic customers could use the advantages of large target market by spending more budget to advertise new products.
9. It is tried to provide a new diffusion model by considering pricing, advertising, and strategic behavior of customers. It is a suitable starting point to provide more reliable diffusion models by investigating other decisions such as warranty.

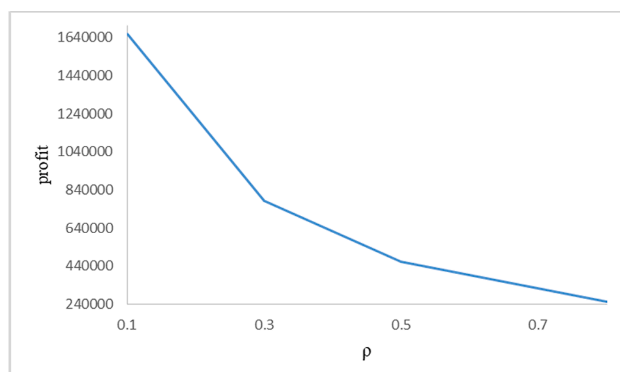

Fig. 5. Effect of the discount rate on profit.

Table 7
Impact of discount rate on price and advertising expenditure.

Parameter	ρ = 0.1		ρ = 0.3		ρ = 0.5		ρ = 0.8	
Period	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*
1	571.71	14396.72	559.65	18848.77	555.47	20507.78	553.05	21495.25
2	572.99	12647.88	561.38	16156.9	556.94	17571.53	554.18	18471.42
3	573.89	11153.22	562.84	13815.98	558.22	14974.42	555.18	15748.82
4	574.38	9877.56	564.06	11791.47	559.31	12699.22	556.05	13332.36
5	574.43	8792.46	565.02	10051.55	560.24	10723.91	556.80	11215.67
6	573.98	7874.96	565.72	8567.72	561.02	9023.55	557.43	9383.49
7	572.93	7106.71	566.09	7315.67	561.63	7572.49	557.95	7814.22
8	571.14	6473.45	566	6276.56	562.06	6347.15	558.39	6482.45
9	568.40	5964.73	565.19	5439.42	562.18	5330.91	558.74	5361.12
10	564.36	5573.98	563.14	4805.31	561.57	4524.66	559.01	4423.54
11	558.52	5298.81	558.82	4396.49	558.96	3975.20	559.08	3654.31
12	550	5141.66	550	4278.54	550	3873.79	550	3564.67

Table 8
Impact of population size on price and advertising expenditure.

Parameter	N = 10000		N = 20000		N = 30000		N = 40000		N = 50000	
Period	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*	W_1^*	U_1^*
1	558.27	6525.20	559.16	12739.97	559.65	18848.77	560	24890.88	560.26	30884.42
2	560.02	5638.78	560.89	10952.98	561.38	16156.9	561.72	21291.55	561.98	26375.81
3	561.51	4861.22	562.36	9394.03	562.84	13815.98	563.17	18168.44	563.43	22470.4
4	562.77	4182.92	563.59	8041.53	564.06	11791.47	564.38	15473.43	564.62	19106.09
5	563.78	3594.98	564.58	6875.52	565.02	10051.55	565.34	13162.35	565.57	16225.93
6	564.52	3089.34	565.29	5878.09	565.72	8567.725	566.02	11195.68	566.25	13779.06
7	564.93	2659.21	565.67	5033.95	566.09	7315.67	566.38	9539.65	566.60	11721.97
8	564.89	2299.43	565.60	4331.39	565.99	6276.56	566.28	8168.02	566.49	10020.77
9	564.15	2007.38	564.81	3763.82	565.19	5439.42	565.45	7065.06	565.65	8654.79
10	562.23	1784.41	562.81	3332.71	563.14	4805.31	563.38	6231.22	563.55	7623.60
11	558.17	1638.94	558.58	3053.62	558.82	4396.49	558.98	5695.10	559.11	6961.94
12	550	1593.73	550	2970.97	550	4278.54	550	5543.12	550	6776.81

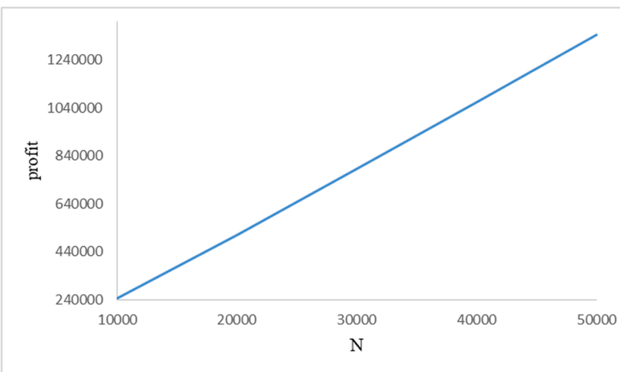

Fig. 6. Effect of the population size on profit.

7. Conclusion

In this paper, a new pricing-advertising diffusion model for generation product was presented. It was assumed that customers were strategic, i.e. they bought the product when they gained the maximum surplus. We applied the Pontryagin's maximum principle to solve the proposed model. When being faced with strategic customers, firms' managers can get benefit from the results of this research in setting and adapting dynamic pricing and advertising strategies to gain higher profit. It was shown that there was a threshold for the optimal advertising path and the optimal pricing path was a concave curve. The findings of our research revealed that firms should determine the lower price and advertising expenditure in the markets with strategic customers. Based our results, the firms' profit will be damaged by strategic customers. It is beneficial for the firms when customers are myopic and /or their strategic behavior is high. Results highlighted that the price would be increased by increasing the effect of the WOM advertising and the firm's advertising. When the impact of advertising effectiveness, market's population size, and discount rate is high, it is better to increase the advertising expenditure. The advertising expenditure reduces with increasing WOM advertising effectiveness. Firms to gain more profit should be more patient in markets populated with strategic customers.

The following suggestions are provided to improve the proposed model. We modeled the strategic customers' surplus as a function of price and customer evaluation of the product. Considering other factors such as the performance improvement of the next generation will make the problem more realistic. Also, the proposed model will be more practical by considering other marketing variables such as warranty duration. In this research, we considered the length of the planning horizon a constant parameter. It will be valuable to investigate it as a decision variable.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

With deriving from the functions in Equation (15) with respect to time and applying Equations (14), we obtained:

$$\dot{W}_1(t) = \frac{1}{2}[-\rho\lambda(t) + (V_1 - \alpha V_2)(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})^2(\varphi - \phi - \beta Ln(U_1(t)) - 2\varphi\frac{S(t)}{N})]$$

At t = T, we have $(\varphi - \phi - \beta Ln(U_1(t)) - 2\varphi\frac{S(t)}{N}) < 0$ and $\lambda(T) = 0$ (see Kalish, 1983). Therefore, $W_1(T) < 0$.

Considering $\bar{t}$ as the point, at which $(\varphi - \phi - \beta Ln(U_1(\bar{t})) - 2\varphi\frac{S(\bar{t})}{N}) = 0$, $W_1(\bar{t})$ is as follows:

$$\dot{W}_1(\bar{t}) = -\frac{1}{2}\rho\lambda(\bar{t})$$

By assuming that the sign of $W_1(t)$ will not change more than once, we have:

i. $W_1(\bar{t}) > 0$ if $-\rho\lambda(\bar{t}) > 0$; the optimal pricing trajectory is a concave curve.
ii. $W_1(\bar{t}) < 0$ if $-\rho\lambda(\bar{t}) < 0$. Thus, if the sign of $W_1(t)$ varies, it will happen before $\bar{t}$. We spouse $\tilde{t}$as the point that the sign of $W_1(t)$ varies in it. Thus, the second derivative of $W_1(t)$ at $W_1(\tilde{t}) = 0$ is:

$$\ddot{W}_1(\tilde{t})|_{W_1(\tilde{t})=0} = -\varphi(V_1 - \alpha V_2)(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})^2\frac{s(t)}{N} < 0$$

Thus, the optimal pricing trajectory is a concave curve in this situation.

The maximum of optimal pricing trajectory happens at $W_1(t) = 0$. In other words, we have:

$$W_1(t) = -\frac{\rho\lambda(t)}{S_{S(t)}} + V_1 - \alpha(V_2 - W_2(T))$$

Appendix B

With deriving from the functions in Equation (16) with respect to time and using Equation (14), we gained:

$$\frac{\dot{U}_1(t)}{U_1(t)} = \frac{\beta}{U_1(t)}\rho\lambda(t)(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})(N - S(t)) -$$

$$(1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})(\varphi - \phi - \beta Ln(U_1(t)) - 2\varphi\frac{S(t)}{N}) - (1 - \frac{W_1(t) - \alpha W_2(T)}{V_1 - \alpha V_2})(\varphi - \phi - \beta Ln(U_1(t)) - 2\varphi\frac{S(t)}{N})$$

We know $U_1(t) > 0$. Thus, the sing of $U_1(t)$ is indicated by $\frac{\beta}{U_1(t)}\rho\lambda(t)(N - S(t)) - \varphi + \varphi\frac{S(t)}{N}$. $U_1(t) > 0$ if $\frac{\beta}{U_1(t)}\rho\lambda(t)(N - S(t)) - \varphi + \varphi\frac{S(t)}{N} > 0$. Otherwise, $U_1(t) < 0$. Solving $\frac{\beta}{U_1(t)}\rho\lambda(t)(N - S(t)) - \varphi + \varphi\frac{S(t)}{N} > 0$ gives us:

$$U_1(t) < \frac{N\beta\rho\lambda(t)}{\varphi}$$

Setting $\overline{U} = \frac{N\beta\rho\lambda(t)}{\varphi}$, we have at any time t:

i. $U_1(t) > 0$ if $U_1(t) < \overline{U}$
ii. $U_1(t) = 0$ if $U_1(t) = \overline{U}$
iii. $U_1(t) < 0$ if $U_1(t) > \overline{U}$

Appendix C

$$\underset{W_1^n, U_1^n \geqslant 0}{Max}\ \pi = \sum_{n=0}^{T-1} e^{-\rho n}((W_1^n - C)s^n - U_1^n)$$

Subject to

$$\Delta S^n = s^n = \left(\phi + \beta Ln\left(U_1^n\right) + \varphi\frac{S^{n-1}}{N}\right)\left(N - S^{n-1}\right)\left(1 - \frac{W_1^n - \alpha W_2^T}{V_1 - \alpha V_2}\right)$$
$$n = 1, 2 \ldots T - 1$$

$$S^0 = 0$$

References

Adams, M. (2004). The PDMA Foundation's Comparative Performance Assessment Study (CPAS) Results". Comparative Performance Assessment Conference, New Orleans.

Aviv, Y., & Pazgal, A. (2008). Optimal pricing of seasonal products in the presence of forward- looking consumers. *Manufacturing & Service Operation Management, 10*(3), 339–359. https://doi.org/10.1287/msom.1070.0183

Bass, F. M. (1969). A new product growth for model consumer durables. *Management science, 15*(5), 215–227. https://doi.org/10.1287/mnsc.15.5.215

Bass, F. M., & Bultez, A. V. (1982). Technical note— A note on optimal strategic pricing of technological innovations. *Marketing Science, 1*(4), 371–378. https://doi.org/10.1287/mksc.1.4.371

Cachon, G. P., & Feldman, P. (2015). Price commitments with strategic consumers: Why it can be optimal to discount more frequently… than optimal. *Manufacturing & Service Operations Management, 17*(3), 399–410. https://doi.org/10.1287/msom.2015.0527

Chen, M., & Chen, Z.-L. (2015). Recent developments in dynamic pricing research: Multiple products, competition, and limited demand information. *Production and Operations Management, 24*(5), 704–731. https://doi.org/10.1111/poms.2015.24.issue-510.1111/poms.12295

Chen, Y., & Trichakis, N. (2021). On Revenue Management with Strategic Customers Choosing When and What to Buy. *Operations Research, 69*(1), 175–187. https://doi.org/10.1287/opre.2020.2008

Dhakal, T., Min, K. S., & Lim, D. E. (2019). Review of multi-generation innovation diffusion models. *Industrial Engineering & Management Systems, 18*(4), 794–807. https://doi.org/10.7232/iems.2019.18.4.794

Dockner, E., & Jørgensen, S. (1988). Optimal advertising policies for diffusion models of new product innovation in monopolistic situations. *Management science, 34*(1), 119–130. https://doi.org/10.1287/mnsc.34.1.119

Dong, J., & Wu, D. D. (2019). Two-period pricing and quick response with strategic customers. *International Journal of Production Economics, 215*, 165–173. https://doi.org/10.1016/j.ijpe.2017.06.007

Elmaghraby, W., Gülcü, A., & Keskinocak, P. (2008). Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing & Service Operations Management, 10*(1), 126–148. https://www.researchgate.net/publication/254595992.

Elmaghraby, W., Lippman, S. A., Tang, C. S., & Yin, R. (2009). Will more purchasing options benefit customers? *Production and Operations Management, 18*(4), 381–401. https://doi.org/10.1111/j.1937-5956.2009.01041.x

Feichtinger, G. (1982). Optimal pricing in a diffusion model with concave price-dependent market potential. *Operations Research Letters, 1*(6), 236–240. https://doi.org/10.1016/0167-6377(82)90028-1

Gönsch, J., Klein, R., Neugebauer, M., & Steinhardt, C. (2013). Dynamic pricing with strategic customers. *Journal of Business Economics, 83*(5), 505–549. https://doi.org/10.1007/s11573- 013-0663-7

Guidolin, M. (2008). *Aggregate and Agent-Based Models for the Diffusion of Innovations.* PhD Thesis. Padova: Università di Padova.

Guo, Z., & Chen, J. (2018). Multigeneration Product Diffusion in the Presence of Strategic Consumers. *Information Systems Research, 29*(1), 206–224. https://doi.org/10.1287/isre.2017.0720

Homer, J. B. (1987). A diffusion model with application to evolving medical technologies. *Technological Forecasting and Social Change, 31*(3), 197–218. https://doi.org/10.1016/0040-1625(87)90011-4

Horsky, D., & Simon, L. S. (1983). Advertising and the diffusion of new products. *Marketing Science, 2*(1), 1–17. https://doi.org/10.1287/mksc.2.1.1

Huang, H. Z., Liu, Z. J., & Murthy, D. N. P. (2007). Optimal reliability, warranty and price for new products. *Iie Transactions, 39*(8), 819–827. https://doi.org/10.1080/07408170601091907

Kapur, P. K., Sachdeva, N., & Singh, O. (2017). Optimal profit for manufacturers in product remanufacturing diffusion dynamics. *Journal of Industrial and Production Engineering, 34*(8), 568–579. https://doi.org/10.1080/21681015.2017.1295405

Kalish, S. (1983). Monopolist pricing with dynamic demand and production cost. *Marketing Science, 2*(2), 135–159. https://doi.org/10.1287/mksc.2.2.135

Krishnan, T. V., & Jain, D. C. (2006). Optimal dynamic advertising policy for new products. *Management science, 52*(12), 1957–1969. https://doi.org/10.1287/mnsc.1060.0585

Krishnan, T. V., Bass, F. M., & Jain, D. C. (1999). Optimal pricing strategy for new products. *Management science, 45*(12), 1650–1663. https://doi.org/10.1287/mnsc.45.12.1650

Lee, C. Y., & Huh, S. Y. (2017). Technology forecasting using a diffusion model incorporating replacement purchases. *Sustainability, 9*(6), 1038. https://doi.org/10.3390/su9061038

Li, J., Granados, N., & Netessine, S. (2014). Are consumers strategic? Structural estimation from the air-travel industry. *Management science, 60*(9), 2114–2137. https://doi.org/10.1287/mnsc.2013.1860

Liu, X., Lin, K., Wang, L., & Ding, L. (2020). Pricing decisions for a sustainable supply chain in the presence of potential strategic customers. *Sustainability, 12*(4), 1655. https://doi.org/10.3390/su12041655

Liu, J., Zhai, X., & Chen, L. (2018). The interaction between product rollover strategy and pricing scheme. *International Journal of Production Economics, 201*, 116–135. https://doi.org/10.1016/j.ijpe.2018.03.027

Liu, J., Zhai, X., & Chen, L. (2019). Optimal pricing strategy under trade-in program in the presence of strategic consumers. *Omega, 84*, 1–17. https://doi.org/10.1016/j.omega.2018.03.005

Liang, C., Çakanyıldırım, M., & Sethi, S. P. (2014). Analysis of product rollover strategies in the presence of strategic customers. *Management Science, 60*(4), 1033–1056. https://doi.org/10.1287/mnsc.2013.1803

Lobel, I., Patel, J., Vulcano, G., & Zhang, J. (2015). Optimizing product launches in the presence of strategic consumers. *Management science, 62*(6), 1778–1799. https://doi.org/10.1287/mnsc.2015.2189

Mahajan, V., Muller, E., & Bass, F. M. (1995). Diffusion of new products: Empirical generalizations and managerial uses. *Marketing science, 14*(3), 79–88. https://doi.org/10.1287/mksc.14.3.G79

Mahajan, V., & Peterson, R. A. (1978). Innovation diffusion in a dynamic potential adopter population. *Management Science, 24*(15), 1589–1597. https://doi.org/10.1287/mnsc.24.15.1589

Mahajan, V., & Peterson, R. A. (1982). Erratum to Innovation Diffusion in a Dynamic Potential Adopter Population. *Management Science, 28*(9), 1087. https://doi.org/10.1287/mnsc.28.9.108

Osadchiy, N., & Bendoly, E. (2015). Are consumers really strategic? *Implications from an experimental study*. https://doi.org/10.2139/ssrn.2593184

Özer, ̈O., Ozer, O., & Phillips, R., (2012). The Oxford Handbook of Pricing Management. Oxford University Press.

Rogers, E. M. (1983). *Diffusion of Innovations* (3rd ed.). London: Free Press.

Rosen, J. (1968). Numerical solution of optimal control problems. *Mathematics of Decision Science, Part-2. American Mathematical Society*, 37–45.

Sale, R. S., Mesak, H. I., & Inman, R. A. (2017). A dynamic marketing-operations interface model of new product updates. *European Journal of Operational Research, 257*(1), 233–242. https://doi.org/10.1016/j.ejor.2016.07.051

Şeref, M. M., Carrillo, J. E., & Yenipazarli, A. (2016). Multi-generation pricing and timing decisions in new product development. *International Journal of Production Research, 54*(7), 1919–1937. https://doi.org/10.1080/00207543.2015.1061220

Sethi, S. P., Prasad, A., & He, X. (2008). Optimal advertising and pricing in a new-product adoption model. *Journal of Optimization Theory and Applications, 139*(2), 351–360. https://doi.org/10.1007/s10957-008-9472-5

Sethi, S. P., & Thompson, G. L. (2000). *Optimal control theory: Applications to management science and economics*. Springer, US, Boston, MA: Kluwer Academic.

Shi, X., Fernandes, K., & Chumnumpan, P. (2014). Diffusion of multi-generational high-technology products. *Technovation, 34*(3), 162–176. https://doi.org/10.1016/j.technovation.2013.11.008

Shum, S., Tong, S., & Xiao, T. (2017). On the impact of uncertain cost reduction when selling to strategic customers. *Management Science, 63*(3), 843–860. https://doi.org/10.1287/mnsc.2015.2355

Swami, S., & Dutta, A. (2010). Advertising strategies for new product diffusion in emerging markets: Propositions and analysis. *European Journal of Operational Research, 204*(3), 648–661. https://doi.org/10.1016/j.ejor.2009.11.009

Swami, S., & Khairnar, P. J. (2006). Optimal normative policies for marketing of products with limited availability. *Annals of Operations Research, 143*(1), 107–121. https://doi.org/10.1007/s10479-006-7375-0

Swinney, R. (2011). Selling to strategic consumers when product value is uncertain: The value of matching supply and demand. *Management Science, 57*(10), 1737–1751. https://doi.org/10.1287/mnsc.1110.1360

Tang, C. S., & Netessine, S. (Eds.) (2009). Consumer-driven demand and operations management models: a systematic study of information-technology-enabled sales mechanisms. Springer-Verlag US.

Wei, M. M., & Zhang, F. (2018). Recent research developments of strategic consumer behavior in operations management. *Computers & Operations Research, 93*, 166–176. https://doi.org/10.1016/j.cor.2017.12.005

Wright, M., Upritchard, C., & Lewis, T. (1997). A validation of the Bass new product diffusion model in New Zealand. Marketing Bulletin-Department of Marketing Massey University, 8, 15-29.

Wu, Xiang, (Yale) Gong, Yeming, Xu, Haoxuan, Chu, Chengbin, & Zhang, Jinlong (2017). Dynamic lot-sizing models with pricing for new products. *European Journal of Operational Research, 260*(1), 81–92. https://doi.org/10.1016/j.ejor.2016.12.008

Yenipazarli, A. (2015). A road map to new product success: Warranty, advertisement and price. *Annals of Operations Research, 226*(1), 669–694. https://doi.org/10.1007/s10479-014-1650-2

Zhou, Chuiri, & Wu, Yu (2011). Study on revenue management considering strategic customer behavior. *Journal of Service Science and Management, 04*(04), 507–512. https://doi.org/10.4236/jssm.2011.44058

Zhou, E., Zhang, J., Gou, Q., & Liang, L. (2015). A two period pricing model for new fashion style launching strategy. *International Journal of Production Economics, 160*, 144–156. https://doi.org/10.1016/j.ijpe.2014.10.008